

## **Some Environmental Pollution Control Models**

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### **ABSTRACT:**

The present paper provides some models for environmental pollution control. Here we have formed and discussed mathematical models mainly for controlling pollution of air, noise and waste products. We have also given here causes of environmental pollution and its consequences on human health and life expectancy and mental well-being.

**Key Words:** Environment, Pollution, control, waste product model

### **1. Introduction:**

Environmental pollution has emerged as one of the most pressing challenges of the 21<sup>st</sup> century [1,2,4,5]. The issue has led to threats to human health, ecosystems and the overall well-being of our planet. Infact environmental pollution refers to the introduction of harmful materials into the environment. In other words, environmental pollution is the contamination to such an extent that normal environmental processes are adversely affected. Various types of models can be framed for environmental pollution based on different parameters. Such type of models can be categorized as follows [3, 6-8]:

- (i) Water pollution control models
- (ii) Air pollution control models
- (iii) Control models for the disposal of solid wastes
- (iv) Noise pollution control models
- (v) Total environment pollution control models
- (vi) Global environment pollution control models.

Global environment pollution control models.

In an earlier paper by me, I have already discussed water pollution control models in the context of pollution of rivers. Similar pollution control models can be developed for pollution control of oceans, lakes, and ponds. Sometimes, huge amounts of waste products are dumped into oceans and lakes and these lead to the destruction of fish on a large scale. The diffusion of these products poses enormous and difficult mathematical problems whose

solution is important for knowing the area affected. In some villages, the use of a common pond for bathing of humans and animals, for washing of clothes and for taking drinking water may pose important health hazards to the community.

Here we shall mainly discuss on air pollution on control models, control models for solid waste disposal and noise pollution control models.

Air pollution can be caused by point sources (e.g., smoke coming out of factory chimneys) or line sources (e.g., a continuous line of cars on a road emitting petroleum fumes) or area sources (due to fermentation of biological waste products from dumping grounds). Pollutants such as sulphur dioxide emitted from these sources are diffused in the air by wind velocity and are inhaled by all living beings in these areas. It is therefore essential for the owners of these sources to reduce the outputs of the pollutants, but this costs money. Optimal air pollution control models are aimed at minimizing these costs subject to attaining a certain quality of pollutant-free life in the affected area.

The emergence of modern industrial societies has led to the accumulation of waste amounts of are products which have to be dumped on and, and in rives, lakes, and oceans. The economic disposal of these waste products with minimum cost and minimum pollution is important.

Noise effects from factories, cars aeroplanes, and so on affect quality of life significantly. Noise pollution control therefore aims at minimizing the output of noise at sources and the input of noise at critical receiving points.

By dumping waste products in rivers and lakes or by burning them, we may minimize the cost of their disposal, but this may result in water and air pollution. Hence the concept of total environment control becomes important. Our aim is to simultaneously control air and noise pollution in regions by taking their interactions into account.

The pollution control measures of one country many cause pollution increase in a neighbouring country through diffusion of pollutants in the atmosphere and the oceans. Thus we need models for global pollution control.

## **2. Formation of Models**

### **(I) Air Pollution Control Models**

Let

$N$  = number of sources of emission of pollution,

$K$  = number of pollutants emitted by each source,

$S(n)$  = number of control states of the  $n$ -th source ( $n = 1, 2, \dots, N$ ),

$C(n,s)$  = cost of keeping the  $n$ -th source in the  $s$ -th state ( $s = 1, 2, \dots, S(n)$ )

$E(n,s,k)$  = emission of the  $k$ -th pollutant when the  $n$ -th source is in the  $s$ -th control state ( $k = 1, 2, \dots, K$ ),

$M$  = number of receptor points in the area of interest,

$A(n, s, k, m)$  = concentration of the  $k$ -th pollutant at the  $n$ -th receptor due to emission from the  $n$ -th source in the  $s$ -th state ( $m = 1, 2, \dots, M$ ).

The elements  $A(n, s, k, m)$  have to be calculated by using appropriate diffusion models.

Our first air pollution control model requires that costs must be minimized subject to the total amount of emission of the  $k$ -th pollutant not exceeding a specified amount  $B(k)$ . this gives the model:

$$(2.1) \text{ Minimize } \sum_{n=1}^N \sum_{s=1}^{S(n)} c(n,s)X(n,s)$$

subject to

$$(2.2) \sum_{n=1}^N \sum_{s=1}^{S(n)} E(n,s,k)X(n,s) \leq B(k), \quad k = 1, 2, \dots, K,$$

$$(2.3) \sum_{n=1}^{S(n)} X(n,s) = 1 \quad n = 1, 2, \dots, N,$$

where  $X(n, s)$  can take the value zero or one only. It is only one when the  $n$ -th source is in the  $s$ -th state and is zero if the  $n$ -th source is in any other state. The decision variable is  $X(n, s)$ , i.e., the state in which each source has to be.

Our second air pollution control model requires that the amount of the  $k$ -th pollutant received at any receptor be less than or equal to a specified amount  $C(k)$ . This means replacing (2.2) by

$$(2.4) \sum_{n=1}^N \sum_{s=1}^{S(n)} A(n, s, k, m) X(n, s) \leq C(k), \quad n = 1, 2, \dots, M;$$

$$k = 1, 2, \dots, K$$

This model was used for New York AQCR with 678 point sources and 863 area sources ( $N = 1541$ ), with 278 reception locations ( $M = 278$ ) for two pollutants, viz, sulphur dioxide and particulates ( $K = 2$ ), with  $C(1) = 80 \mu\text{g}/\text{m}^3$  for  $\text{SO}_2$  and  $C(2) = 75 \mu\text{g}/\text{m}^3$  for particulates.

## (II) Control Models for Solid Waste Disposal

One problem is to consider the locations of a number of landfills which receive solid wastes from a series of waste generation sources. The problem may be formulated as follows. Given : (i) the location of sources, (ii) the amounts of wastes generated at each source, (iii) possible landfill capacity limitations, and (iv) a set of transportation costs. Determine (a) the number of landfills, (b) the location of each landfill, (c) the allocation of sources to landfills, and (d) amounts to be transported. If there is a single landfill of unlimited capacity, the problem reduces to Weber's problem of minimizing.

$$(2.5) \sum_{j=1}^n w_j [(x - x_j)^2 + (y - y_j)^2]^{1/2}$$

Sometimes when we are allowed to move along rectangular road only, we have to minimize

$$(2.6) \sum_{j=1}^n w_j (|x - x_j| + |y - y_j|)$$

The solutions of these problems are available. If  $w_j = w$  in (4.5.6), the answer is the median point.

If the sources and landfill locations are fixed, the problem is similar to the transportation problem.

$$(2.7) \text{Minimize } \sum_{j=1}^n \sum_{i=1}^m f_{ij}(x_{ij})$$

subject to

$$(2.8) \sum_{j=1}^n x_{ij} = a_i, \quad \sum_{i=1}^m x_{ij} \leq b_j \quad (x_{ij} \geq 0)$$

where,

$i = 1, 2, \dots, m$  denotes the  $m$  waste product sources,

$j = 1, 2, \dots, n$  denotes the  $n$  landfills,

$a_i$  is the amount of waste products at the  $i$ -th source,

$b_j$  is the capacity of the  $j$ -th landfill,

$$(2.9) \quad \sum_{j=1}^n b_j \geq \sum_{i=1}^m a_i$$

If  $f_{ij}(x_{ij}) = c_{ij} x_{ij}$ , we can use in some cases the transportation technique. If fixed costs are involved and

$$(2.10) \quad f_{ij}(x_{ij}) = k_{ij} \delta_{ij} + c_{ij} x_{ij},$$

where  $\delta_{ij} = 0$  if  $x_{ij} = 0$  and  $\delta_{ij} = 1$  if  $x_{ij} > 0$ , then we can use the fixed-charge transportation techniques.

### (III) Noise Pollution Control Model

The decibel noise level resulting from a noise  $Q$  is given by

$$(2.11) \quad B = 10 \log_{10} (Q/Q_0)$$

where  $Q_0$  is a certain reference level.

Let  $B_i$  denote the pressure level of sound at a unit distance from the source at site  $i$ .

Then the sound pressure at the receptor  $j$  at a distance  $d_{ij}$  from the source  $i$  is given by

$$(2.12) \quad 10^{B_{ij}/10} = \frac{10^{B_i/10}}{d_{ij}^2}$$

so that the sound pressure at the receptor  $j$  due to all noise sources is

$$(2.13) \quad B_j = 10 \log_{10} \sum_i 10^{B_i/10}$$

Let  $X_i^s, X_j^r$  be the reduction in the noise pressure at the source site  $i$  and the receptor site  $j$ ;

let the associated costs be  $C_i^s(X_i^s)$  and  $C_j^r(X_j^r)$ . Also, let the cost of any noise attenuation

device  $S_h$  between  $i$  and  $j$  be  $C_h^s(S_h)$ , and let the resulting reductions in noise pressure be

$A_{ij}^h(S_h)$ . Then our optimization model is

$$(2.14) \quad \text{Minimize} \quad \sum_i C_i^s(X_i^s) + \sum_j C_j^r(X_j^r) + \sum_h C_h(S_h)$$

subject to

$$(2.15) \quad 10 \log_{10} \sum_i (\ln 10) \exp(k_{ij}) - X_j^r < (B_j)_{\max}$$

$$(2.16) \quad X_i^s > 0, X_j^r > 0, S_h > 0 \text{ for all } i, j, h$$

where

$$k_{ij} = [B_{ij} - X_i^s - \sum_h A_{ij}^h(S_h)]/10$$

and  $(B_j)_{\max}$  is the maximum permissible noise pressure at the receptor j.

### 3. Causes of Environmental Pollution

The causes of environmental pollutions are based on the following factors.

- Rapid industrialization
- Rapid urbanization
- Forest fires
- Improper agricultural practices
- De forestation

**Other causes:** Other prominent causes of environmental pollution include continued reliance on fossil fuels, vehicular emissions and improper waste management.

### 4. Discussion:

Environmental pollution has severe consequences on human health, ecosystems and the planet. Below are five key effects.

- (i) Impact on human health.
- (ii) Damage of ecosystems and bio diversity.
- (iii) Climate change acceleration
- (iv) Economic consequences
- (v) Degradation of quality of life.

In fact excessive noise, light and air pollution lower life expectancy and mental well-being. Polluted cities experience reduced visibility, foul odors and poor living conditions, making daily life uncomfortable and unhealthy for residents.

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